The Singapore Syllabus

ALIGNED TO THE LATEST 2027 EDITION MOE SYLLABUS

Secondary 4 Mathematics

ASSESSMENT BOOK



- **Notes** with worked examples
- **Practice questions** with marks shown
- **Answers** with full working
- **2 full practice exams** with mark schemes

2 full practice exams **162** questions **251** pages

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Answers and Mark Schemes

Chapters 1 to 4 and both examinations 201

CHAPTER 1

Set Language and Notation

Part 1: Notes and Worked Examples



In the notes

12 ideas. Read each idea and its worked example, then try the Try This question before you check its answer. The practice questions start on page 17.

- Sets and Elements
- The Universal Set, the Empty Set and the Complement
- Subsets
- Intersection and Disjoint Sets
- Union
- Venn Diagrams of Two Sets
- Shading Regions in a Venn Diagram
- Describing a Shaded Region
- Venn Diagrams of Three Sets
- Problems on the Number of Elements: Two Sets
- Problems on the Number of Elements: Three Sets
- Formulas to Remember

Name: _____ Class: _____ Date: _____

SECONDARY 4 · MATHEMATICS · SET LANGUAGE AND NOTATION

Set Language and Notation

Theory and worked examples

Sets and Elements

A set is a well-defined collection of distinct objects. The objects in a set are called its elements (or members). A set is usually named with a capital letter, and its elements are written inside curly brackets { }. The order of the elements does not matter, and each element is written only once.

- Listing the elements: A = {2, 3, 5, 7}.
- Describing the elements in words: V = {vowels of the English alphabet}.
- Set-builder notation: C = {x : x is an integer, $1 \le x \le 6$}. Read it as "the set of all x such that x is an integer from 1 to 6". So C = {1, 2, 3, 4, 5, 6}.
- $x \in A$ means "x is an element of A". $x \notin A$ means "x is not an element of A". For A above, $5 \in A$ and $9 \notin A$.
- n(A) is the number of elements in A. For A = {2, 3, 5, 7}, n(A) = 4.
- Two sets are equal when they have exactly the same elements: {1, 4, 9} = {9, 1, 4}.

WORKED EXAMPLE

P = {x : x is a prime number, $10 < x < 30$}. (a) List the elements of P. (b) Find n(P). (c) Is 21 an element of P? Write your answer using $\in$ or $\notin$.

1. (a) The prime numbers between 10 and 30 are 11, 13, 17, 19, 23 and 29. So P = {11, 13, 17, 19, 23, 29}.
2. (b) P has 6 elements, so n(P) = 6.
3. (c) $21 = 3 \times 7$, so 21 is not prime. Hence $21 \notin P$.

(a) P = {11, 13, 17, 19, 23, 29} (b) n(P) = 6 (c) $21 \notin P$

TRY THIS

Q = {letters of the word GEOGRAPHY}. List the elements of Q and find n(Q).

Answer: Q = {G, E, O, R, A, P, H, Y}, n(Q) = 8 (G is written once)

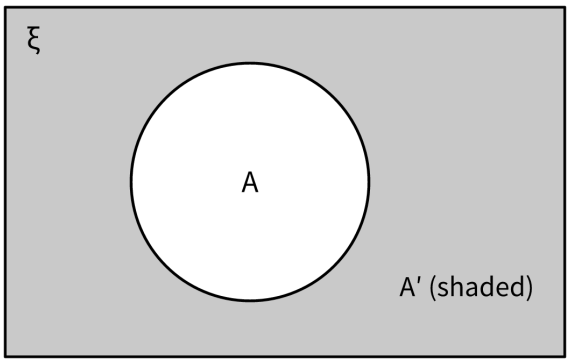
The Universal Set, the Empty Set and the Complement

The universal set, written ξ, is the set of all the elements being considered in a question. Every other set in the question is a subset of ξ. In a Venn diagram, ξ is drawn as a rectangle. The empty set, written $\varnothing$ or $\{\}$, is the set with no elements, so $n(\varnothing) = 0$. The complement of A, written A', is the set of elements of ξ that are not in A.

KEY IDEA

The shaded region in the diagram is A': everything inside the rectangle ξ but outside the circle A.

- Every element of ξ is either in A or in A', never both. So $n(A) + n(A') = n(\xi)$.
- The complement depends on ξ. If ξ changes, A' changes too.
- Do not write the empty set as $\{0\}$ or $\{\varnothing\}$. Each of these is a set with one element.



WORKED EXAMPLE

$\xi = \{x : x \text{ is an integer, } 1 \le x \le 12\}$, $A = \{x : x \text{ is a multiple of 3}\}$ and $B = \{x : x \text{ is a multiple of 13}\}$. (a) List the elements of A'. (b) Find $n(A')$. (c) Describe the set B.

1. (a) $A = \{3, 6, 9, 12\}$. A' is every other element of ξ: $A' = \{1, 2, 4, 5, 7, 8, 10, 11\}$.
2. (b) $n(A') = 8$. Check: $n(A) + n(A') = 4 + 8 = 12 = n(\xi)$.
3. (c) No integer from 1 to 12 is a multiple of 13, so B has no elements. $B = \varnothing$.

(a) $A' = \{1, 2, 4, 5, 7, 8, 10, 11\}$ (b) 8 (c) $B = \varnothing$, the empty set

TRY THIS

$\xi = \{x : x \text{ is an integer, } 20 \le x \le 30\}$ and $S = \{x : x \text{ is a perfect square}\}$. Find $n(S')$.

Answer: $n(\xi) = 11$ and $S = \{25\}$, so $n(S') = 11 - 1 = 10$

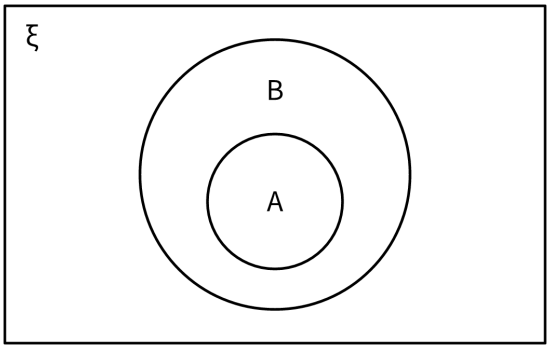
Subsets

A is a subset of B, written $A \subseteq B$, if every element of A is also an element of B. A is a proper subset of B, written $A \subset B$, if $A \subseteq B$ and B has at least one element that is not in A (so $A \neq B$).

KEY IDEA

The diagram shows $A \subset B$: every element of A lies inside B, and B has room for elements that are not in A.

- $A \not\subseteq B$ means A is not a subset of B: at least one element of A is not in B.
- $A \not\subset B$ means A is not a proper subset of B.
- Every set is a subset of itself ($A \subseteq A$), but no set is a proper subset of itself.
- The empty set is a subset of every set.
- $\in$ links an element to a set; $\subseteq$ links a set to a set. Write $3 \in \{1, 3, 5\}$ but $\{3\} \subseteq \{1, 3, 5\}$.
- In a Venn diagram, $A \subset B$ is drawn with the circle A completely inside the circle B.



WORKED EXAMPLE

$S = \{2, 4, 6, 8\}$, $T = \{x : x \text{ is an even number}, 1 \leq x \leq 9\}$ and $U = \{4, 8, 12\}$. State whether each statement is true or false. (a) $S \subseteq T$ (b) $S \subset T$ (c) $U \not\subseteq S$ (d) $\{4, 8\} \subset U$

1. List T first: $T = \{2, 4, 6, 8\}$, so S and T have exactly the same elements and $S = T$.
2. (a) Every element of S is in T, so $S \subseteq T$ is true.
3. (b) T has no element that is not in S, so S is not a proper subset of T. $S \subset T$ is false.
4. (c) $12 \in U$ but $12 \notin S$, so U is not a subset of S. $U \not\subseteq S$ is true.
5. (d) 4 and 8 are both in U, and U also has 12. So $\{4, 8\} \subset U$ is true.

(a) True (b) False (c) True (d) True

TRY THIS

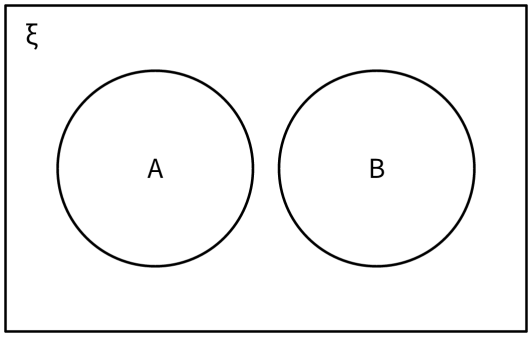
List all the subsets of {p, q}. Which of them are proper subsets of {p, q}?

Answer: $\varnothing$, {p}, {q}, {p, q}. The proper subsets are $\varnothing$, {p} and {q}.

Intersection and Disjoint Sets

The intersection of A and B, written $A \cap B$, is the set of elements that are in both A and B. In a Venn diagram it is the region where the two circles overlap. If A and B have no elements in common, $A \cap B = \varnothing$ and the sets are disjoint. Disjoint sets are drawn as circles that do not touch, as in the diagram.

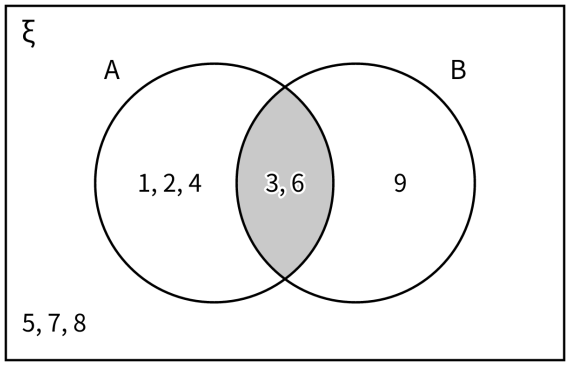
- $A \cap B$ is part of A and part of B, so $n(A \cap B)$ can never be more than $n(A)$ or $n(B)$.
- If $A \subseteq B$, then $A \cap B = A$.



WORKED EXAMPLE

ξ = {x : x is an integer, $1 \leq x \leq 9$}, A = {x : x is a factor of 12} and B = {x : x is a multiple of 3}. (a) List the elements of $A \cap B$. (b) Draw a Venn diagram to show ξ, A and B.

1. A = {1, 2, 3, 4, 6} (12 is not in ξ) and B = {3, 6, 9}.
2. (a) The elements in both lists are 3 and 6. So $A \cap B = \{3, 6\}$.
3. (b) Write 3 and 6 in the overlap first. Then 1, 2 and 4 go in A only, 9 goes in B only, and 5, 7 and 8 go outside both circles.



4. The shaded overlap is $A \cap B$.

(a) $A \cap B = \{3, 6\}$ (b) see the Venn diagram

Name: _____ Class: _____ Date: _____

SECONDARY 4 · MATHEMATICS · SET LANGUAGE AND NOTATION

Set Language and Notation

INSTRUCTIONS

- 21 questions, 59 marks in total.
- Answer all questions in the spaces provided.
- Show your working clearly. Marks are given for correct working.

Section A: Short Questions

1 $\xi = \{x : x \text{ is an integer}, 1 \leq x \leq 15\}$ and $A = \{x : x \text{ is a multiple of } 4\}$.

(a) List the elements of A.

Ans: _____ [1]

(b) Find $n(A')$.

Ans: _____ [1]

2 $M = \{x : x \text{ is a prime number}, 30 < x < 50\}$.

(a) List the elements of M.

Ans: _____ [1]

(b) Write down $n(M)$.

Ans: _____ [1]

3 P = {2, 5, 8, 11}. State whether each of these statements is true or false.

(a) $\{5\} \in P$

Ans: _____ [1]

(b) $\{2, 11\} \subset P$

Ans: _____ [1]

4 $n(\xi) = 24$ and $n(A') = 9$. Find $n(A)$.

Ans: _____ [1]

5 $\xi = \{a, b, c, d, e, f, g\}$, $X = \{a, c, e, g\}$ and $Y = \{b, c, d\}$. List the elements of

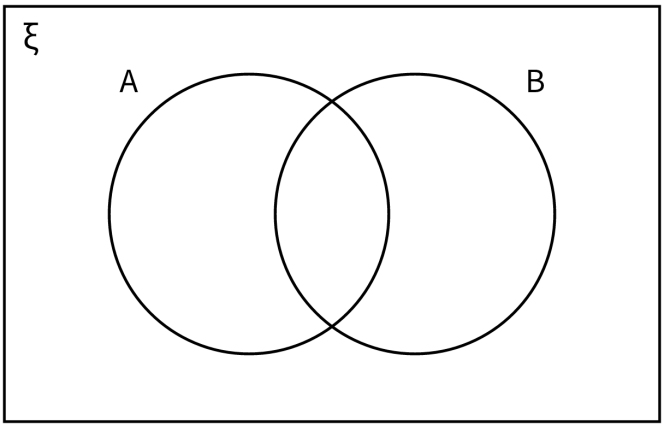
(a) $X \cap Y$,

Ans: _____ [1]

(b) $X' \cap Y'$.

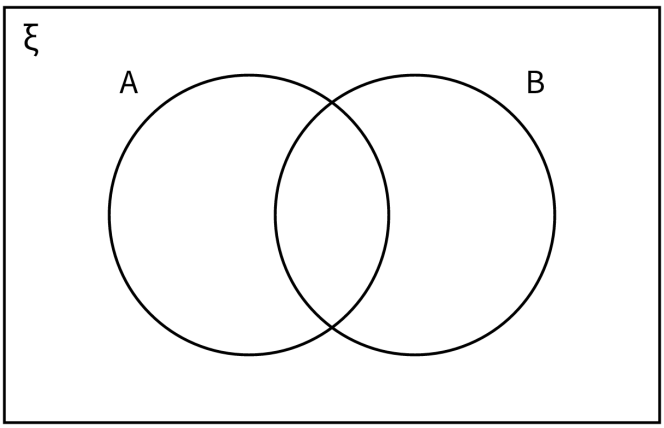
Ans: _____ [1]

6 On the Venn diagram, shade the region which represents $A \cap B'$.



[1]

7 On the Venn diagram, shade the region which represents $(A' \cap B)'$.



[1]

Chapter 1: Set Language and Notation (Answer Key)

21 questions, 59 marks. M: method mark, A: answer mark, B: mark for the answer alone.

1. (a) {4, 8, 12} (b) 12 **[2]**

(a) A = {4, 8, 12} [1]

B1 {4, 8, 12}

(b) n(A′) = 15 − 3 = 12 [1]

B1 12

2. (a) {31, 37, 41, 43, 47} (b) 5 **[2]**

(a) M = {31, 37, 41, 43, 47} [1]

B1 all five primes, no others

(b) n(M) = 5 [1]

B1 5 (follow through from (a))

3. (a) False (b) True **[2]**

(a) False: {5} is a set, not an element. The correct statements are $5 \in P$ or $\{5\} \subset P$. [1]

B1 False

(b) True: 2 and 11 are in P, and P has other elements (5 and 8). [1]

B1 True

4. 15 **[1]**

B1 n(A) = 24 − 9 = 15

5. (a) {c} (b) {f} **[2]**

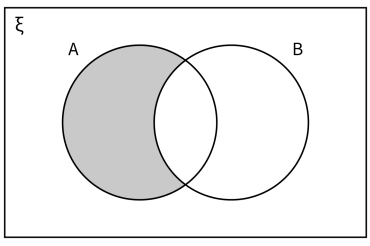
(a) $X \cap Y = \{c\}$ [1]

B1 {c}

(b) $X' = \{b, d, f\}$ and $Y' = \{a, e, f, g\}$, so $X' \cap Y' = \{f\}$ [1]

B1 {f}

6. Shade the region in A but not in B (A only). **[1]**



B1 A only shaded, nothing else