The Singapore Syllabus

ALIGNED TO THE LATEST 2027 EDITION MOE SYLLABUS

Secondary 4 Additional Mathematics

ASSESSMENT BOOK



- **Notes** with worked examples
- **Practice questions** with marks shown
- **Answers** with full working
- **2 full practice exams** with mark schemes

2 full practice exams **232** questions **421** pages

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CHAPTER 1

Addition and Double Angle Formulae

Part 1: Notes and Worked Examples



In the notes

11 ideas. Read each idea and its worked example, then try the Try This question before you check its answer. The practice questions start on page 18.

- The Addition Formulae
- Exact Values of Other Angles
- Using the Formulae with Given Ratios
- Angles in a Diagram
- The Double Angle Formulae
- Squares and Half Angles
- Simplifying Trigonometric Expressions
- Solving Equations with Double Angles
- Equations with the Addition Formulae, in Radians
- Proving Identities with the Formulae
- Formulas to Remember

Name: _____ Class: _____ Date: _____

Addition and Double Angle Formulae

Theory and worked examples

The Addition Formulae

The addition formulae (also called the compound angle formulae) give the sine, cosine and tangent of a sum or a difference of two angles A and B in terms of the ratios of A and B. They hold for all angles, in degrees or in radians. They are given in the formula list of the O-Level paper, but you must know how to use them quickly in both directions: to expand, and to recognise an expansion as a single ratio.

KEY IDEA

Learn the sign patterns: sine keeps the sign, cosine changes the sign, and tangent keeps it on top and changes it below.

- $\sin(A + B)$ is NOT $\sin A + \sin B$. For example, $\sin(30° + 60°) = \sin 90° = 1$, but $\sin 30° + \sin 60° = \frac{1 + \sqrt{3}}{2}$.
- Read a formula backwards to combine: $\sin 3x \cos x - \cos 3x \sin x = \sin(3x - x) = \sin 2x$.
- In the sine formula the order of the terms is sin cos ± cos sin. In the cosine formula it is cos cos ∓ sin sin.

Formula	Sign pattern
$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	same sign as in the bracket
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	opposite sign: cos(A + B) has a minus
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	same sign on top, opposite sign below

WORKED EXAMPLE

(a) Expand and simplify $\sin(\theta + 60°)$. (b) Without using a calculator, find the value of $\cos 50° \cos 10° - \sin 50° \sin 10°$. (c) Write $\frac{\tan 4x + \tan x}{1 - \tan 4x \tan x}$ as a single trigonometric ratio.

1. (a) $\sin(\theta + 60°) = \sin\theta \cos 60° + \cos\theta \sin 60°$
2. $= \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$
3. (b) The pattern cos cos − sin sin is cos(A + B) with A = 50° and B = 10°.
4. $\cos 50° \cos 10° - \sin 50° \sin 10° = \cos(50° + 10°) = \cos 60° = \frac{1}{2}$
5. (c) The pattern is tan(A + B) with A = 4x and B = x, so the expression is $\tan(4x + x) = \tan 5x$.

(a) $\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$ (b) $\frac{1}{2}$ (c) $\tan 5x$

TRY THIS

Without using a calculator, find the exact value of $\sin 100° \cos 55° - \cos 100° \sin 55°$.

Answer: $\sin(100° - 55°) = \sin 45° = \frac{\sqrt{2}}{2}$

Exact Values of Other Angles

Angles such as 15°, 75°, 105° and 165° can be written as a sum or a difference of the special angles 30°, 45°, 60° (and 90°, 180°). The addition formulae then give their exact values in surd form. In radians, $\frac{\pi}{12} = 15°$, $\frac{5\pi}{12} = 75°$ and $\frac{7\pi}{12} = 105°$.

KEY IDEA

Writing $\sin 45° = \frac{\sqrt{2}}{2}$ (not $\frac{1}{\sqrt{2}}$) keeps the denominators rational: $\sin 45° \cos 30° = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{4}$.

- $15° = 45° - 30°$ or $60° - 45°$; $75° = 45° + 30°$; $105° = 60° + 45°$; $165° = 120° + 45°$.
- $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ and $\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$.
- Use the exact values $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 45° = \cos 45° = \frac{\sqrt{2}}{2}$, $\tan 30° = \frac{\sqrt{3}}{3}$, $\tan 60° = \sqrt{3}$.
- Give the answer in simplest surd form with a rational denominator, as in the chapter Surds.

WORKED EXAMPLE

Without using a calculator, find the exact values of (a) sin 75°, (b) tan 15°.

1. (a) $\sin 75° = \sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30°$
2. $= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$
3. $= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$
4. (b) $\tan 15° = \tan(45° - 30°) = \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} = (1 - \frac{\sqrt{3}}{3}) \div (1 + \frac{\sqrt{3}}{3})$
5. Multiply the numerator and the denominator by 3: $= \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$
6. $= \frac{(3 - \sqrt{3})(3 - \sqrt{3})}{(3 + \sqrt{3})(3 - \sqrt{3})} = \frac{9 - 6\sqrt{3} + 3}{9 - 3} = \frac{12 - 6\sqrt{3}}{6}$
7. $= 2 - \sqrt{3}$

(a) $\frac{\sqrt{6} + \sqrt{2}}{4}$ (b) $2 - \sqrt{3}$

TRY THIS

Find the exact value of $\cos \frac{7\pi}{12}$.

Answer: $\cos(\frac{\pi}{3} + \frac{\pi}{4}) = \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$

Using the Formulae with Given Ratios

A common question gives one ratio of A and one ratio of B, with the quadrant of each angle, and asks for a ratio of A + B or A − B without finding A and B. First find sin, cos and tan of each angle, as in Secondary 3: draw the angle in its quadrant, complete the right-angled triangle with Pythagoras' theorem, and give each ratio its sign by ASTC.

KEY IDEA

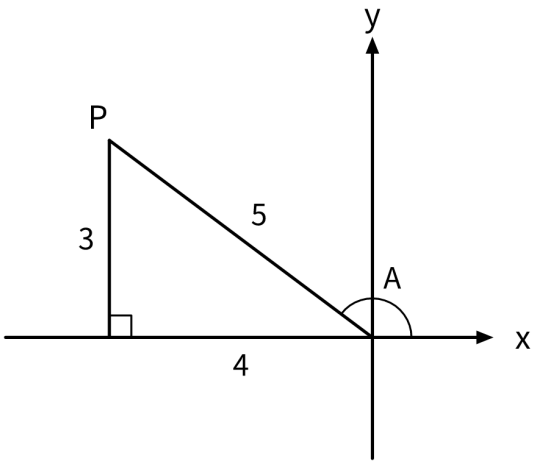
Put every negative ratio in brackets when you substitute. Most lost marks here are sign errors.

- Step 1: For each angle, sketch the quadrant and the triangle, and find the missing side.
- Step 2: Write down sin, cos and tan of each angle, with signs from ASTC.
- Step 3: Substitute into the correct formula, with brackets around negative values.
- Step 4: Simplify the fraction. The sign of the answer narrows A + B to two quadrants.

WORKED EXAMPLE

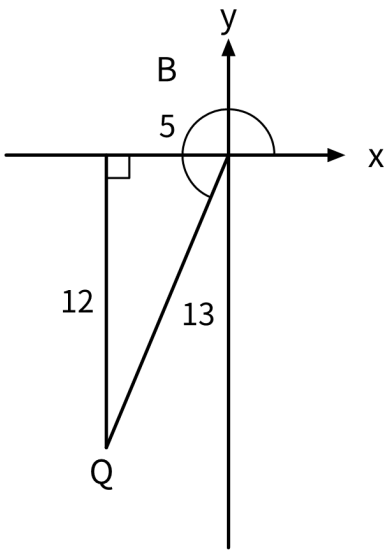
Given that $\sin A = \frac{3}{5}$, where A is obtuse, and $\tan B = \frac{12}{5}$, where $180° < B < 270°$, find, without using a calculator, the exact values of (a) cos(A + B), (b) tan(A − B).

1. A is obtuse, so A is in the 2nd quadrant: draw the triangle with sides 3, 4 and 5.



2. $\sin A = \frac{3}{5}$, $\cos A = -\frac{4}{5}$, $\tan A = -\frac{3}{4}$

3. $180° < B < 270°$, so B is in the 3rd quadrant and $\tan B = \frac{12}{5} = \frac{-12}{-5}$: the triangle has sides 5, 12 and 13.



4. $\sin B = -\frac{12}{13}$, $\cos B = -\frac{5}{13}$

5. (a) $\cos(A + B) = \cos A \cos B - \sin A \sin B = (-\frac{4}{5})(-\frac{5}{13}) - (\frac{3}{5})(-\frac{12}{13})$

6. $= \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$

7. (b) $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = (-\frac{3}{4} - \frac{12}{5}) \div (1 + (-\frac{3}{4})(\frac{12}{5}))$

8. $= (-\frac{63}{20}) \div (-\frac{16}{20}) = \frac{63}{16}$

(a) $\frac{56}{65}$ (b) $\frac{63}{16}$

Name: _____ Class: _____ Date: _____

SECONDARY 4 · ADDITIONAL MATHEMATICS · ADDITION FORMULAE, DOUBLE ANGLE FORMULAE, EQUATION...

Addition and Double Angle Formulae

INSTRUCTIONS

- 18 questions, 75 marks in total.
- Answer all questions in the spaces provided.
- Show your working clearly. Marks are given for correct working.

Section A: Short Questions

1 Express each of the following as a single trigonometric ratio.

(a) $\sin 52° \cos 22° - \cos 52° \sin 22°$

Ans: _____ [1]

(b) $2 \sin 4\theta \cos 4\theta$

Ans: _____ [1]

(c) $\cos 3x \cos x + \sin 3x \sin x$

Ans: _____ [1]

2 Without using a calculator, find the exact value of $\cos 75°$, in simplest surd form.

Ans: _____ [2]

3 Without using a calculator, show that $\tan 105° = -2 - \sqrt{3}$.

[3]

4 Given that $\tan A = 3$, find the exact value of $\tan(A + 45°)$.

Ans: _____ [2]

5 Show that $\sin(x + 30°) + \cos(x + 60°) \equiv \cos x$.

[2]

Chapter 1: Addition and Double Angle Formulae (Answer Key)

18 questions, 75 marks. M: method mark, A: answer mark, B: mark for the answer alone.

1. (a) $\sin 30°$ (b) $\sin 8\theta$ (c) $\cos 2x$ **[3]**

(a) $\sin(52° - 22°) = \sin 30° \left(= \frac{1}{2}\right)$ [1]

B1 $\sin 30°$

(b) $\sin 2(4\theta) = \sin 8\theta$ [1]

B1 $\sin 8\theta$

(c) $\cos(3x - x) = \cos 2x$ [1]

B1 $\cos 2x$

2. $\frac{\sqrt{6} - \sqrt{2}}{4}$ **[2]**

M1 $\cos(45° + 30°) = \cos 45° \cos 30° - \sin 45° \sin 30° = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}$

A1 $\frac{\sqrt{6} - \sqrt{2}}{4}$

3. $\tan(60° + 45°) = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} = \frac{(\sqrt{3} + 1)(1 + \sqrt{3})}{(1 - \sqrt{3})(1 + \sqrt{3})} = \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$ (shown) **[3]**

M1 $\tan(60° + 45°) = \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$

A1 $\frac{\sqrt{3} + 1}{1 - \sqrt{3}}$

A1 rationalise with $1 + \sqrt{3}$ to reach $-2 - \sqrt{3}$

4. -2 **[2]**

M1 $\frac{\tan A + \tan 45°}{1 - \tan A \tan 45°} = \frac{3 + 1}{1 - 3}$

A1 -2

5. LHS $= \sin x \cos 30° + \cos x \sin 30° + \cos x \cos 60° - \sin x \sin 60° = \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x + \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x = \cos x =$ RHS **[2]**

M1 both expansions correct

A1 exact values substituted and simplified to cos x

6. $\frac{\sqrt{5}}{3}$ **[2]**

M1 $2\cos^2\theta - 1 = \frac{1}{9}$, so $\cos^2\theta = \frac{5}{9}$

A1 $\cos\theta = \frac{\sqrt{5}}{3}$ (positive, as θ is acute)

7. $\sin^2 3x = \frac{1}{2} - \frac{1}{2} \cos 6x \left(a = \frac{1}{2}, b = -\frac{1}{2}\right)$ **[2]**

M1 $\cos 6x = 1 - 2\sin^2 3x$

A1 $a = \frac{1}{2}, b = -\frac{1}{2}$