The Singapore Syllabus

ALIGNED TO THE LATEST 2027 EDITION MOE SYLLABUS

Secondary 3 Mathematics

ASSESSMENT BOOK



- **Notes** with worked examples
- **Practice questions** with marks shown
- **Answers** with full working
- **A full practice exam** with mark scheme

9 chapters **330** pages **232** questions

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CHAPTER 1

Numbers and their Operations

Part 1: Notes and Worked Examples



In the notes

11 ideas. Read each idea and its worked example, then try the Try This question before you check its answer. The practice questions start on page 14.

- Index Notation and the Laws for the Same Base
- Laws for the Same Index
- Zero and Negative Indices
- Fractional Indices and Roots
- Simplifying to Positive Index Form
- Evaluating with the Laws of Indices
- Solving Equations by Equating Indices
- Standard Form
- Prefixes and Large Numbers in Words
- Calculating in Standard Form
- Formulas to Remember

Name: _____ Class: _____ Date: _____

SECONDARY 3 · MATHEMATICS · INDICES AND STANDARD FORM

Numbers and their Operations

Theory and worked examples

Index Notation and the Laws for the Same Base

In a^n, a is the base and n is the index (or power). For a positive whole number n, a^n means n copies of a multiplied together: $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. When two powers have the same base, three laws let us simplify without writing out every factor.

KEY IDEA

These laws work only when the bases are the same. $2^3 \times 5^2$ cannot be written as a single power.

- Law 1 (multiply: add the indices): $a^m \times a^n = a^{m+n}$. For example, $x^4 \times x^3 = x^7$.
- Law 2 (divide: subtract the indices): $a^m \div a^n = a^{m-n}$. For example, $y^9 \div y^2 = y^7$.
- Law 3 (power of a power: multiply the indices): $(a^m)^n = a^{mn}$. For example, $(p^3)^4 = p^{12}$.
- The numbers in front (the coefficients) are multiplied or divided as usual: $3x^4 \times 5x^2 = 15x^6$.

WORKED EXAMPLE

Simplify $6p^7q^3 \times 2p^2q \div 4p^5q^2$.

1. Deal with the numbers first: $6 \times 2 \div 4 = 3$.
2. Powers of p: $p^{7+2-5} = p^4$ (Laws 1 and 2).
3. Powers of q: q is q^1, so $q^{3+1-2} = q^2$.
4. Put them together: $3p^4q^2$.

$3p^4q^2$

TRY THIS

Simplify $(y^4)^3 \div y^5$.

Answer: y^7

Laws for the Same Index

When two numbers are raised to the same index, the index can be shared out over a product or a quotient. A power outside a bracket applies to everything inside the bracket, including the number in front.

KEY IDEA

There is no law for adding or subtracting powers. $a^m + a^n$ is NOT a^{m+n}, and $a^n + b^n$ is NOT $(a + b)^n$. Check: $2^3 + 2^3 = 16$, but $2^6 = 64$.

- Law 4: $(ab)^n = a^n b^n$. For example, $(2x^3)^4 = 2^4 \times x^{12} = 16x^{12}$, not $2x^{12}$.
- Law 5: $(\frac{a}{b})^n = \frac{a^n}{b^n}$, where $b \neq 0$. For example, $(\frac{x}{3})^2 = \frac{x^2}{9}$.
- A negative number inside the bracket: an even index gives a positive answer and an odd index gives a negative answer. $(-2)^4 = 16$ but $(-2)^3 = -8$.

WORKED EXAMPLE

Simplify $(\frac{3x^2}{y^3})^3$.

1. Raise every part to the power 3 (Laws 4 and 5).
2. Number: $3^3 = 27$.
3. $(x^2)^3 = x^6$ and $(y^3)^3 = y^9$ (Law 3).
4. So $(\frac{3x^2}{y^3})^3 = \frac{27x^6}{y^9}$.

$$\frac{27x^6}{y^9}$$

TRY THIS

Simplify $(-2a^3b)^5$.

Answer: $-32a^{15}b^5$

Zero and Negative Indices

Law 2 tells us what a zero index and a negative index mean. Since $a^3 \div a^3 = 1$ and also $a^3 \div a^3 = a^{3-3} = a^0$, we get $a^0 = 1$. Since $a^0 \div a^n = a^{0-n}$, we get $a^{-n} = \frac{1}{a^n}$. A negative index means 'one over', the reciprocal. It does not make the number negative.

KEY IDEA

A negative index moves a factor across the fraction line: x^{-4} on top becomes x^4 underneath, and $\frac{1}{y^{-2}} = y^2$.

- $a^0 = 1$ for any $a \neq 0$: $7^0 = 1$, $(-3)^0 = 1$ and $(5x)^0 = 1$ $(x \neq 0)$.
- $a^{-n} = \frac{1}{a^n}$: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.
- A fraction to a negative index: turn it upside down and make the index positive. $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$, so $(\frac{2}{5})^{-2} = (\frac{5}{2})^2 = \frac{25}{4}$.
- Watch what the index belongs to: $3x^{-2} = \frac{3}{x^2}$ (only x has the index), but $(3x)^{-2} = \frac{1}{9x^2}$.

WORKED EXAMPLE

Evaluate $4^{-2} + 3^0 \times (\frac{1}{2})^{-3}$.

1. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
2. $3^0 = 1$
3. $(\frac{1}{2})^{-3} = 2^3 = 8$
4. $\frac{1}{16} + 1 \times 8 = 8\frac{1}{16}$

$8\frac{1}{16}$

TRY THIS

Evaluate $(\frac{3}{4})^{-2}$.

Answer: $\frac{16}{9} = 1\frac{7}{9}$

Fractional Indices and Roots

Since $(a^{\frac{1}{2}})^2 = a^1 = a$, the index $\frac{1}{2}$ means square root. In the same way $a^{\frac{1}{n}}$ is the nth root of a. For $a > 0$, $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$: the denominator of the index is the root and the numerator is the power. Take the root first; the numbers stay small.

KEY IDEA

Read $a^{-\frac{m}{n}}$ in three steps: the minus sign means reciprocal, n means take the nth root, m means raise to the power m.

- $a^{\frac{1}{2}} = \sqrt{a}$ and $a^{\frac{1}{3}} = \sqrt[3]{a}$. For example, $25^{\frac{1}{2}} = 5$ and $64^{\frac{1}{3}} = 4$.
- $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$. For example, $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$.
- With a negative sign as well, take the reciprocal: $32^{-\frac{3}{5}} = \frac{1}{(\sqrt[5]{32})^3} = \frac{1}{2^3} = \frac{1}{8}$.
- A root can be written as an index, and the other way round: $\sqrt{x^5} = x^{\frac{5}{2}}$ and $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$.

WORKED EXAMPLE

Evaluate $81^{-\frac{3}{4}}$ without using a calculator.

1. The minus sign: $81^{-\frac{3}{4}} = \frac{1}{81^{\frac{3}{4}}}$.
2. The 4 means fourth root: $\sqrt[4]{81} = 3$, since $3 \times 3 \times 3 \times 3 = 81$.
3. The 3 means cube it: $3^3 = 27$.
4. So $81^{-\frac{3}{4}} = \frac{1}{27}$.

$\frac{1}{27}$

TRY THIS

Express $\sqrt[5]{y^2}$ as a power of y.

Answer: $y^{\frac{2}{5}}$

Name: _____ Class: _____ Date: _____

SECONDARY 3 · MATHEMATICS · INDICES AND STANDARD FORM

Numbers and their Operations

INSTRUCTIONS

- 24 questions, 61 marks in total.
- Answer all questions in the spaces provided.
- Show your working clearly. Marks are given for correct working.

Section A: Short Questions

1 Evaluate $49^{-\frac{1}{2}}$.

Ans: _____ [1]

2 Write $\sqrt[4]{y^3}$ as a power of y.

Ans: _____ [1]

3 Express 0.000 038 5 in standard form.

Ans: _____ [1]

4 A music video was watched 26.4 million times. Write this number in standard form.

Ans: _____ [1]

5 Simplify $4m^5n^2 \times 3m^{-2}n^4$.

Ans: _____ [1]

6 The wavelength of a red light is 6.8×10^{-7} m. Express this wavelength in nanometres.

Ans: _____ nm [1]

7 Without using a calculator,

(a) write 4.7×10^{120} in the form $k \times 10^{121}$,

Ans: _____ [1]

(b) hence find $4.7 \times 10^{120} + 3.2 \times 10^{121}$, giving your answer in standard form.

Ans: _____ [1]

8 Evaluate $(\frac{2}{3})^{-3} + 5^0$, giving your answer as a mixed number.

Ans: _____ [2]

Section B: Structured Questions

9 Simplify $\left(\frac{125p^9}{q^{-6}}\right)^{-\frac{1}{3}}$, giving your answer in positive index form.

Ans: _____ [2]

10 Simplify $\left(\frac{16x^{-8}}{9y^4}\right)^{-\frac{3}{2}}$, giving your answer in positive index form.

Ans: _____ [2]

11 Simplify $\sqrt[3]{-8a^6b^{-9}}$, giving your answer in positive index form.

Ans: _____ [2]

Chapter 1: Numbers and their Operations (Answer Key)

24 questions, 61 marks. M: method mark, A: answer mark, B: mark for the answer alone.

1. $\frac{1}{7}$ [1]

B1 $\frac{1}{7}$

2. $y^{\frac{3}{4}}$ [1]

B1 $y^{\frac{3}{4}}$

3. 3.85×10^{-5} [1]

B1

4. 2.64×10^{7} [1]

B1

5. $12m^3n^6$ [1]

B1

6. 680 nm [1]

B1 6.8×10^{-7} m = 680×10^{-9} m = 680 nm

7. (a) 0.47×10^{121} (b) 3.67×10^{121} [2]

(a) $4.7 \times 10^{120} = 0.47 \times 10^{121}$

B1

(b) $0.47 \times 10^{121} + 3.2 \times 10^{121} = 3.67 \times 10^{121}$

B1

8. $4\frac{3}{8}$ [2]

M1 $(\frac{2}{3})^{-3} = (\frac{3}{2})^3 = \frac{27}{8}$

A1 $\frac{27}{8} + 1 = 4\frac{3}{8}$

9. $\frac{1}{5p^3q^2}$ [2]

M1 $125^{-\frac{1}{3}} = \frac{1}{5}$, or turning the fraction over: $(\frac{q^{-6}}{125p^9})^{\frac{1}{3}}$

A1 $\frac{q^{-2}}{5p^3} = \frac{1}{5p^3q^2}$