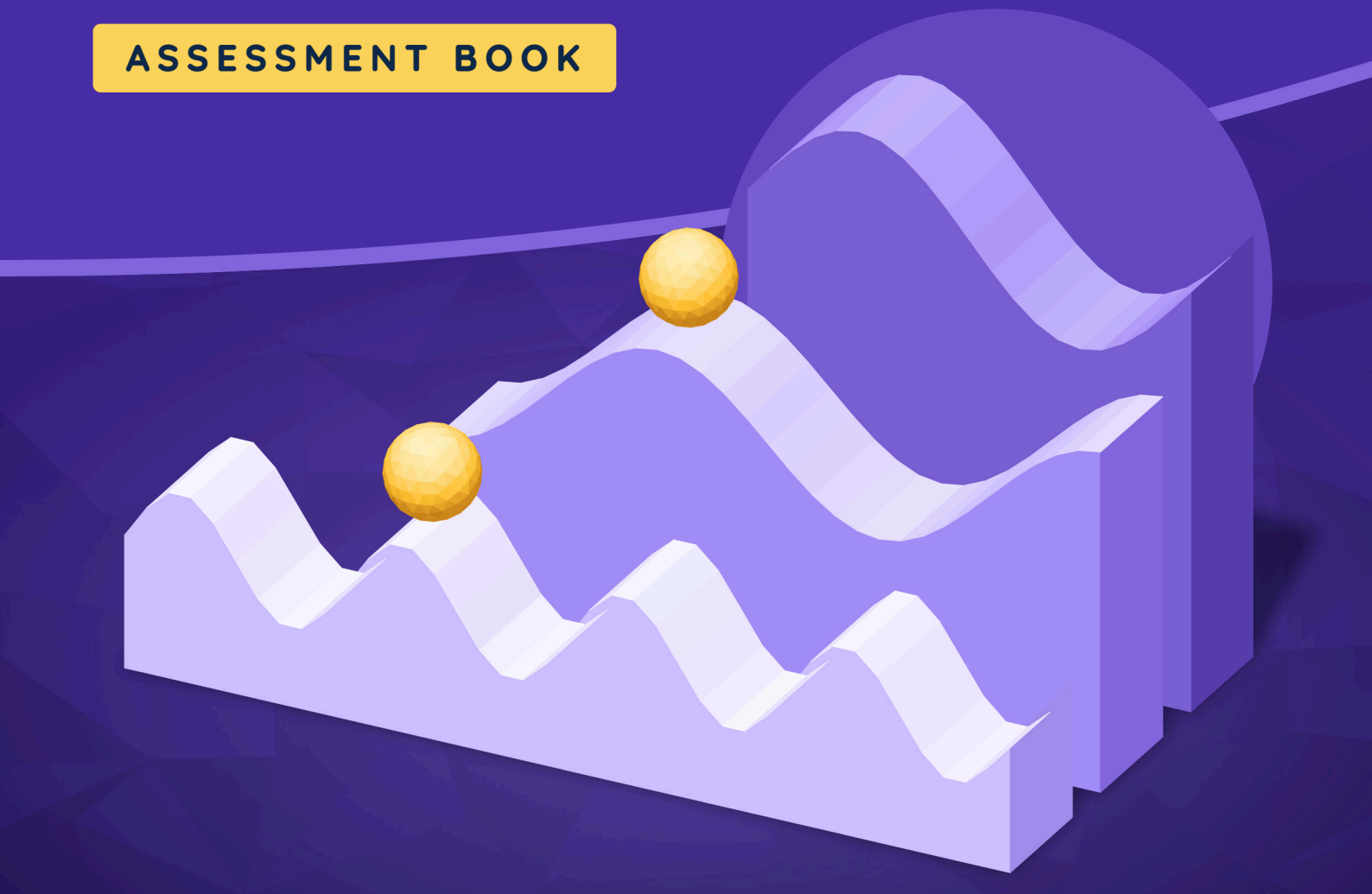
The Singapore Syllabus

ALIGNED TO THE LATEST 2027 EDITION MOE SYLLABUS

Secondary 3 Additional Mathematics

ASSESSMENT BOOK



- **Notes** with worked examples
- **Answers** with full working
- **Practice questions** with marks shown
- **A full practice exam** with mark scheme

12 chapters **409** pages **249** questions

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CHAPTER 1

Quadratic Functions

Part 1: Notes and Worked Examples



In the notes

11 ideas. Read each idea and its worked example, then try the Try This question before you check its answer. The practice questions start on page 16.

- The Graph of $y = ax^2 + bx + c$
- Completing the Square
- Maximum and Minimum Values
- The Turning Point from the Factorised Form
- Sketching a Quadratic Curve
- Sketching over a Given Interval
- Finding the Equation of a Quadratic Curve
- Always Positive or Always Negative: Completing the Square
- Always Positive or Always Negative: Using a and $b^2 - 4ac$
- Quadratic Functions as Models
- Formulas to Remember

Name: _____ Class: _____ Date: _____

SECONDARY 3 · ADDITIONAL MATHEMATICS · QUADRATIC FUNCTIONS

Quadratic Functions

Theory and worked examples

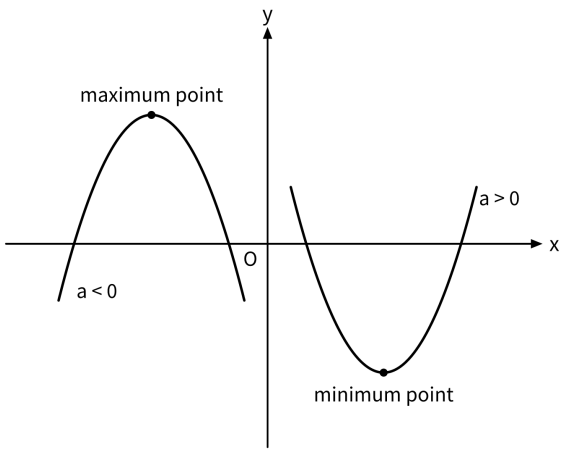
The Graph of $y = ax^2 + bx + c$

A quadratic function has the form $y = ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$. Its graph is a smooth curve called a parabola. The sign of a decides the shape of the curve.

KEY IDEA

Look at the sign of a first. It tells you whether the turning point is a minimum point or a maximum point.

- If $a > 0$, the curve opens upwards (a U shape) and has a minimum point.
- If $a < 0$, the curve opens downwards (an ∩ shape) and has a maximum point.
- The minimum or maximum point is called the turning point.
- y-intercept: put $x = 0$, so $y = c$. The curve cuts the y-axis at (0, c).
- x-intercepts: put $y = 0$ and solve $ax^2 + bx + c = 0$. There may be two, one or none.
- The curve is symmetrical about the vertical line through its turning point (the line of symmetry). The two x-intercepts are the same distance from this line.

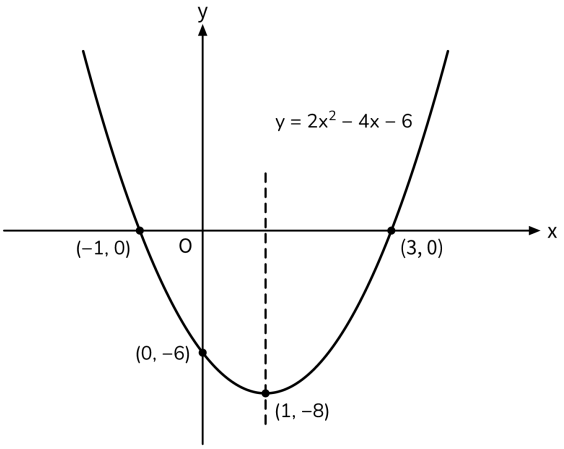


WORKED EXAMPLE

For the curve $y = 2x^2 - 4x - 6$, state whether the turning point is a maximum or a minimum point. Find the intercepts on the axes and the equation of the line of symmetry.

1. $a = 2 > 0$, so the curve opens upwards and the turning point is a minimum point.

2. When $x = 0$, $y = -6$. The y-intercept is $(0, -6)$.
3. When $y = 0$: $2x^2 - 4x - 6 = 0$, so $2(x^2 - 2x - 3) = 0$ and $2(x - 3)(x + 1) = 0$.
4. $x = 3$ or $x = -1$. The x-intercepts are $(-1, 0)$ and $(3, 0)$.
5. The line of symmetry is halfway between the x-intercepts: $x = \frac{-1 + 3}{2} = 1$.



Minimum point; (0, −6), (−1, 0) and (3, 0); x = 1

TRY THIS

For $y = -x^2 + 6x - 5$, state whether the turning point is a maximum or a minimum point, and find the intercepts on the axes.

Answer: $a = -1 < 0$, so a maximum point. $y = -(x - 1)(x - 5)$: $(0, -5)$, $(1, 0)$ and $(5, 0)$; the line of symmetry is $x = 3$.

Completing the Square

Every quadratic expression $ax^2 + bx + c$ can be written in the form $a(x - h)^2 + k$, where h and k are constants. This is called completing the square. As in Secondary 3 Mathematics, for $a = 1$ we add and subtract the square of half the coefficient of x.

KEY IDEA

When a is negative, the signs inside the bracket change: $-3x^2 + 12x = -3(x^2 - 4x)$. When you multiply back, $-3 \times (-4)$ gives $+ 12$.

- $a = 1$: $x^2 + px + q = (x + \frac{p}{2})^2 + q - (\frac{p}{2})^2$. For example, $x^2 - 6x + 2 = (x - 3)^2 - 9 + 2 = (x - 3)^2 - 7$.
- $a \neq 1$: take a out of the x^2 term and the x term only, not the constant: $3x^2 + 12x + 5 = 3(x^2 + 4x) + 5$.
- Complete the square inside the bracket, then multiply back by a: $3[(x + 2)^2 - 4] + 5 = 3(x + 2)^2 - 12 + 5 = 3(x + 2)^2 - 7$.
- Check by expanding: $3(x + 2)^2 - 7 = 3(x^2 + 4x + 4) - 7 = 3x^2 + 12x + 5$.

WORKED EXAMPLE

Express $-2x^2 + 12x - 7$ in the form $a(x - h)^2 + k$.

1. Take -2 out of the first two terms: $-2x^2 + 12x - 7 = -2(x^2 - 6x) - 7$
2. Complete the square inside the bracket: $x^2 - 6x = (x - 3)^2 - 9$
3. $-2[(x - 3)^2 - 9] - 7 = -2(x - 3)^2 + 18 - 7$
4. $= -2(x - 3)^2 + 11$
5. Check: $-2(x^2 - 6x + 9) + 11 = -2x^2 + 12x - 18 + 11 = -2x^2 + 12x - 7$

$-2(x - 3)^2 + 11$

TRY THIS

Express $3x^2 + 9x + 2$ in the form $a(x + p)^2 + q$.

Answer: $3(x^2 + 3x) + 2 = 3[(x + \frac{3}{2})^2 - \frac{9}{4}] + 2 = 3(x + \frac{3}{2})^2 - \frac{19}{4}$

Maximum and Minimum Values

A square is never negative: $(x - h)^2 \geq 0$ for every real value of x, and $(x - h)^2 = 0$ only when $x = h$. So the form $y = a(x - h)^2 + k$ shows the turning point at once.

KEY IDEA

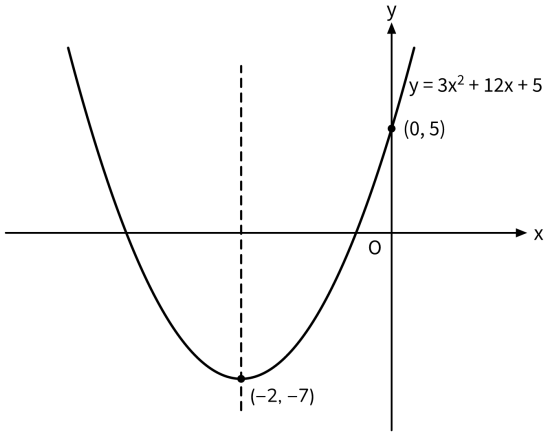
The maximum or minimum value is the y-value k. The value of x at which it happens is h. Give both when the question asks for them.

- If $a > 0$, then $a(x - h)^2 \geq 0$, so $y \geq k$. The minimum value of y is k, when $x = h$.
- If $a < 0$, then $a(x - h)^2 \leq 0$, so $y \leq k$. The maximum value of y is k, when $x = h$.
- The turning point is (h, k), and the line of symmetry is $x = h$.
- Watch the sign of h: $y = 2(x + 3)^2 - 5 = 2(x - (-3))^2 - 5$, so $h = -3$ and the minimum point is $(-3, -5)$.

WORKED EXAMPLE

Find the minimum value of $y = 3x^2 + 12x + 5$ and the corresponding value of x. Hence write down the coordinates of the turning point.

1. $y = 3(x^2 + 4x) + 5 = 3[(x + 2)^2 - 4] + 5 = 3(x + 2)^2 - 7$
2. $(x + 2)^2 \geq 0$ for all real x, so $3(x + 2)^2 \geq 0$ and $y \geq -7$.
3. $y = -7$ when $x + 2 = 0$, that is, when $x = -2$.
4. The minimum value of y is -7, when $x = -2$. The turning point is $(-2, -7)$.



Minimum value −7 when x = −2; turning point (−2, −7)

TRY THIS

Find the maximum value of $y = 5 + 8x - 2x^2$ and the corresponding value of x.

Answer: $y = -2(x^2 - 4x) + 5 = -2(x - 2)^2 + 13$; maximum value 13 when x = 2

The Turning Point from the Factorised Form

If $y = a(x - p)(x - q)$, the curve cuts the x-axis at (p, 0) and (q, 0). By symmetry, the turning point lies on the line halfway between them.

KEY IDEA

When the function is already factorised, this is quicker than completing the square.

- Line of symmetry: $x = \frac{p + q}{2}$.
- Substitute this value of x into the equation to find the y-coordinate of the turning point.
- Find each x-intercept by setting each factor equal to 0: in $y = (2x + 1)(3 - x)$, the x-intercepts are $x = -\frac{1}{2}$ and $x = 3$.
- The sign of a, the coefficient of x^2 when expanded, still decides between a maximum and a minimum point: $(2x + 1)(3 - x)$ has x^2 coefficient $2 \times (-1) = -2 < 0$.

WORKED EXAMPLE

Find the coordinates of the turning point of the curve $y = (2x + 2)(x - 5)$, and state whether it is a maximum or a minimum point.

1. x-intercepts: $2x + 2 = 0$ gives $x = -1$, and $x - 5 = 0$ gives $x = 5$.
2. Line of symmetry: $x = \frac{-1 + 5}{2} = 2$
3. When x = 2: $y = (2(2) + 2)(2 - 5) = (6)(-3) = -18$
4. The coefficient of x^2 is $2 \times 1 = 2 > 0$, so the turning point is a minimum point.

Name: _____ Class: _____ Date: _____

SECONDARY 3 · ADDITIONAL MATHEMATICS · QUADRATIC FUNCTIONS

Quadratic Functions

INSTRUCTIONS

- 20 questions, 75 marks in total.
- Answer all questions in the spaces provided.
- Show your working clearly. Marks are given for correct working.

Section A: Short Questions

1 Express $x^2 - 8x + 3$ in the form $(x - h)^2 + k$.

Ans: _____ [2]

2 Express $2x^2 + 6x - 1$ in the form $a(x + p)^2 + q$.

Ans: _____ [3]

3 Write down the maximum value of $y = 7 - 2(x + 1)^2$ and the corresponding value of x.

Ans: _____ [2]

4 Write down the coordinates of the turning point of the curve $y = 3(x - 4)^2 + 5$, and state whether it is a maximum or a minimum point.

Ans: _____ [2]

5 Find the equation of the line of symmetry of the curve $y = (x + 7)(x - 3)$.

Ans: _____ [2]

6 Show that $x^2 + 10x + 30$ is positive for all real values of x.

[2]

7 Find the range of values of c for which $x^2 - 10x + c$ is positive for all real values of x.

Ans: _____ [3]

8 Explain why $-3x^2 + 2x - 4$ is negative for all real values of x.

[2]

Chapter 1: Quadratic Functions (Answer Key)

20 questions, 75 marks. M: method mark, A: answer mark, B: mark for the answer alone.

1. $(x - 4)^2 - 13$ **[2]**

M1 $(x - 4)^2 - 16 + 3$

A1 $(x - 4)^2 - 13$

2. $2(x + \frac{3}{2})^2 - \frac{11}{2}$ **[3]**

M1 $2(x^2 + 3x) - 1$

M1 $2[(x + \frac{3}{2})^2 - \frac{9}{4}] - 1$

A1 $2(x + \frac{3}{2})^2 - \frac{11}{2}$

3. Maximum value 7, when $x = -1$ **[2]**

B1 maximum value 7

B1 $x = -1$

4. (4, 5), a minimum point **[2]**

B1 (4, 5)

B1 minimum point ($a = 3 > 0$)

5. $x = -2$ **[2]**

M1 x-intercepts -7 and 3, so $x = \frac{-7 + 3}{2}$

A1 $x = -2$

6. $x^2 + 10x + 30 = (x + 5)^2 - 25 + 30 = (x + 5)^2 + 5$. Since $(x + 5)^2 \geq 0$ for all real x, $(x + 5)^2 + 5 \geq 5 > 0$. **[2]**

M1 $(x + 5)^2 + 5$

A1 $(x + 5)^2 \geq 0$, so the expression $\geq 5 > 0$

7. $c > 25$ **[3]**

M1 $a = 1 > 0$, so need $b^2 - 4ac < 0$

M1 $(-10)^2 - 4(1)(c) < 0$, $100 - 4c < 0$

A1 $c > 25$

8. $a = -3 < 0$, and $b^2 - 4ac = 2^2 - 4(-3)(-4) = 4 - 48 = -44 < 0$. The curve $y = -3x^2 + 2x - 4$ opens downwards and does not meet the x-axis, so the expression is always negative. **[2]**

B1 $a = -3 < 0$

B1 $b^2 - 4ac = -44 < 0$, with the conclusion