

O-Level A-Math Formula Sheet

O-Level and SEC (G3) · Additional Mathematics



The Singapore Syllabus

PRINTED IN THE PAPER

Formulae the O-Level A-Math paper gives you

In the order SEAB prints them. You do not need to memorise these, but you do need to know when to use each one.

1. ALGEBRA

Quadratic equation For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

Formulas the O-Level A-Math paper does not give you

The syllabus expects you to use these, but the paper does not print them.

QUADRATICS AND POLYNOMIALS

Nature of roots

$b^2 - 4ac > 0$ two real distinct, $= 0$ equal, < 0 no real roots

Line and curve

substitute, then $b^2 - 4ac > 0$ meets at two points, $= 0$ tangent, < 0 no intersection

Always positive

$ax^2 + bx + c > 0$ for all $x \iff a > 0$ and $b^2 - 4ac < 0$

Remainder and factor theorems

$f(x) \div (x - a)$ leaves remainder $f(a)$, $f(a) = 0 \iff (x - a)$ is a factor

Dividing by $(ax - b)$

remainder $= f\left(\frac{b}{a}\right)$

Sum and difference of cubes

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

General term of a binomial expansion

$T_{r+1} = \binom{n}{r} a^{n-r} b^r$

EXPONENTIALS AND LOGARITHMS

Definition

$a^x = b \iff x = \log_a b$

Laws of logarithms

$\log_a xy = \log_a x + \log_a y$, $\log_a \frac{x}{y} = \log_a x - \log_a y$, $\log_a x^n = n \log_a x$

Change of base

$\log_a b = \frac{\log_c b}{\log_c a}$

COORDINATE GEOMETRY

Midpoint

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Parallel and perpendicular lines

$m_1 = m_2$, $m_1 m_2 = -1$

Equation of a circle

$(x - a)^2 + (y - b)^2 = r^2$

Area of a rectilinear figure (shoelace method) vertices taken in order around the figure

$\frac{1}{2} |x_1 y_2 + x_2 y_3 + \dots + x_n y_1 - y_1 x_2 - y_2 x_3 - \dots - y_n x_1|$

General form

$$x^2 + y^2 + 2gx + 2fy + c = 0, \text{ centre } (-g, -f), r = \sqrt{g^2 + f^2 - c}$$

TRIGONOMETRY

Quotient identity

$$\tan A = \frac{\sin A}{\cos A}$$

Exact values

$$\sin 30^\circ = \frac{1}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}, \tan 30^\circ = \frac{1}{\sqrt{3}}, \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}, \tan 45^\circ = 1, \sin 60^\circ = \frac{\sqrt{3}}{2}, \cos 60^\circ = \frac{1}{2}, \tan 60^\circ = \sqrt{3}$$

Reciprocal ratios

$$\sec x = \frac{1}{\cos x}, \operatorname{cosec} x = \frac{1}{\sin x}, \cot x = \frac{1}{\tan x}$$

R-formula

$$a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha), R = \sqrt{a^2 + b^2}, \tan \alpha = \frac{b}{a}$$

$$a \cos \theta \pm b \sin \theta = R \cos(\theta \mp \alpha), R = \sqrt{a^2 + b^2}, \tan \alpha = \frac{b}{a}$$

CALCULUS

Standard derivatives

$$\frac{d}{dx} x^n = nx^{n-1}, \frac{d}{dx} \sin x = \cos x, \frac{d}{dx} \cos x = -\sin x, \frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} e^x = e^x, \frac{d}{dx} \ln x = \frac{1}{x}$$

Chain, product and quotient rules

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}, (uv)' = uv' + vu', \left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

Stationary points

$$\frac{dy}{dx} = 0, \frac{d^2y}{dx^2} > 0 \text{ minimum}, \frac{d^2y}{dx^2} < 0 \text{ maximum}$$

Connected rates of change

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

Standard integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1), \int e^x dx = e^x + c$$

$$\int \sin x dx = -\cos x + c, \int \cos x dx = \sin x + c, \int \sec^2 x dx = \tan x + c$$

Integrals of $(ax + b)$

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c \quad (n \neq -1), \quad \int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

$$\int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + c, \quad \int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + c$$

Area under a curve

$$\text{Area} = \int_a^b y dx$$

Kinematics

$$v = \frac{ds}{dt}, \quad a = \frac{dv}{dt}, \quad s = \int v dt$$