

MF27 List of Formulae (H2 Math)

A-Level H1, H2 and H3 Mathematics · H2 Mathematics



The Singapore Syllabus

PRINTED IN THE PAPER

Formulas MF27 gives you

In the order SEAB prints them. You do not need to memorise these, but you do need to know when to use each one.

ALGEBRAIC SERIES

Binomial expansion where n is a positive integer

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + b^n$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^n}{n!} f^{(n)}(0) + \dots$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$

$|x| < 1$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^r}{r!} + \dots$$

all x

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \dots$$

all x

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^r x^{2r}}{(2r)!} + \dots$$

all x

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + \frac{(-1)^{r+1} x^r}{r} + \dots$$

$-1 < x \leq 1$

PARTIAL FRACTIONS DECOMPOSITION

Non-repeated linear factors

$$\frac{px + q}{(ax + b)(cx + d)} = \frac{A}{(ax + b)} + \frac{B}{(cx + d)}$$

Repeated linear factors

$$\frac{px^2 + qx + r}{(ax + b)(cx + d)^2} = \frac{A}{(ax + b)} + \frac{B}{(cx + d)} + \frac{C}{(cx + d)^2}$$

Non-repeated quadratic factor

$$\frac{px^2 + qx + r}{(ax + b)(x^2 + c^2)} = \frac{A}{(ax + b)} + \frac{Bx + C}{(x^2 + c^2)}$$

TRIGONOMETRY

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2 \cos^2 A - 1 \equiv 1 - 2 \sin^2 A$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

Principal values $|x| \leq 1$

$$-\frac{1}{2}\pi \leq \sin^{-1} x \leq \frac{1}{2}\pi$$

$$0 \leq \cos^{-1} x \leq \pi$$

$|x| \leq 1$

$$-\frac{1}{2}\pi < \tan^{-1} x < \frac{1}{2}\pi$$

DERIVATIVES

$$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$\frac{d}{dx} (\sec x) = \sec x \tan x$$

INTEGRALS

Arbitrary constants are omitted, and a denotes a positive constant, in every row of this table

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right)$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right)$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left(\frac{x-a}{x+a} \right)$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \ln \left(\frac{a+x}{a-x} \right)$$

$$\int \tan x dx = \ln(\sec x)$$

$$\int \cot x dx = \ln(\sin x)$$

$$\int \operatorname{cosec} x dx = -\ln(\operatorname{cosec} x + \cot x)$$

$$\int \sec x dx = \ln(\sec x + \tan x)$$

VECTORS

The point dividing AB in the ratio $\lambda : \mu$ has position vector

$$\frac{\mu \mathbf{a} + \lambda \mathbf{b}}{\lambda + \mu}$$

Vector product

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

APPLICATIONS OF DEFINITE INTEGRALS (NOT IN H2 MATHEMATICS)

Arc length of a curve defined in cartesian form

$$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Surface area of revolution about the x-axis for a curve defined in cartesian form

$$\int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

FUNCTIONS OF TWO VARIABLES (NOT IN H2 MATHEMATICS)

Quadratic approximation of f at (a, b)

$$f(x, y) \approx f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b) + \frac{1}{2}f_{xx}(a, b)(x - a)^2 + f_{xy}(a, b)(x - a)(y - b) + \frac{1}{2}f_{yy}(a, b)(y - b)^2$$

NUMERICAL METHODS (NOT IN H2 MATHEMATICS)

Trapezium rule (for single strip)

$$\int_a^b f(x) dx \approx \frac{1}{2}(b - a) [f(a) + f(b)]$$

Simpson's rule (for two strips)

$$\int_a^b f(x) dx \approx \frac{1}{6}(b - a) \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

The Newton-Raphson iteration for approximating a root of $f(x) = 0$

where x_1 is a first approximation

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Euler Method with step size h

$$y_2 = y_1 + h f(x_1, y_1)$$

Improved Euler Method with step size h

$$u_2 = y_1 + h f(x_1, y_1)$$

$$y_2 = y_1 + \frac{h}{2} [f(x_1, y_1) + f(x_2, u_2)]$$

STANDARD DISCRETE DISTRIBUTIONS

Binomial B(n, p)

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad \text{Mean} = np, \quad \text{Variance} = np(1-p)$$

Poisson Po(λ) not in H2 Mathematics 9758

$$P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}, \quad \text{Mean} = \lambda, \quad \text{Variance} = \lambda$$

Geometric Geo(p) not in H2 Mathematics 9758

$$P(X = x) = (1-p)^{x-1} p, \quad \text{Mean} = \frac{1}{p}, \quad \text{Variance} = \frac{1-p}{p^2}$$

STANDARD CONTINUOUS DISTRIBUTION (NOT IN H2 MATHEMATICS)

Exponential

$$\text{p.d.f.} = \lambda e^{-\lambda x}, \quad \text{Mean} = \frac{1}{\lambda}, \quad \text{Variance} = \frac{1}{\lambda^2}$$

SAMPLING AND TESTING

Unbiased estimate of population variance

$$s^2 = \frac{n}{n-1} \left(\frac{\Sigma(x - \bar{x})^2}{n} \right) = \frac{1}{n-1} \left(\Sigma x^2 - \frac{(\Sigma x)^2}{n} \right)$$

REGRESSION AND CORRELATION

Estimated product moment correlation coefficient

$$r = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\sqrt{\{\Sigma(x - \bar{x})^2\} \{\Sigma(y - \bar{y})^2\}}} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n} \right) \left(\Sigma y^2 - \frac{(\Sigma y)^2}{n} \right)}}$$

Estimated regression line of y on x

$$y - \bar{y} = b(x - \bar{x}), \quad \text{where } b = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\Sigma(x - \bar{x})^2}$$

WILCOXON SIGNED RANK TEST (NOT IN H2 MATHEMATICS)

P = the sum of the ranks corresponding to the positive differences

Q = the sum of the ranks corresponding to the negative differences

T = the smaller of P and Q

Critical values of T

Level of significance for a one-tail (One) or two-tail (Two) test. For each value of n the table gives the largest value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

	One	0.05	0.025	0.01	0.005
	Two	0.1	0.05	0.02	0.01
$n = 6$		2	0		
7		3	2	0	
8		5	3	1	0
9		8	5	3	1
10		10	8	5	3
11		13	10	7	5
12		17	13	9	7
13		21	17	12	9
14		25	21	15	12
15		30	25	19	15
16		35	29	23	19
17		41	34	27	23
18		47	40	32	27
19		53	46	37	32
20		60	52	43	37

MATHEMATICAL RESULTS (NOT IN H2 MATHEMATICS)

AM-GM inequality

For any nonnegative real numbers $x_1, x_2, \dots, x_n$, where the equality holds if and only if $x_1 = x_2 = \dots = x_n$

$$\frac{x_1 + x_2 + \dots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \dots x_n}$$

Cauchy-Schwarz inequality

For any real numbers $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$, where the equality holds if and only if there exists a nonzero constant k such that $u_i = kv_i$ for all $i = 1, 2, \dots, n$

$$\left(\sum_{i=1}^n u_i v_i \right)^2 \leq \left(\sum_{i=1}^n u_i^2 \right) \left(\sum_{i=1}^n v_i^2 \right)$$

Triangle inequality

For any real numbers $x_1, x_2, \dots, x_n$, where the equality holds if $x_1, x_2, \dots, x_n$ are all nonnegative

$$|x_1 + x_2 + \dots + x_n| \leq |x_1| + |x_2| + \dots + |x_n|$$

Inclusion-Exclusion Principle For any subsets $A_1, A_2, \dots, A_n$ of a set

$$\begin{aligned} |A_1 \cup A_2 \cup \dots \cup A_n| &= |A_1| + |A_2| + \dots + |A_n| \\ &\quad - [|A_1 \cap A_2| + |A_1 \cap A_3| + \dots + |A_{n-1} \cap A_n|] \\ &\quad + [|A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + \dots + |A_{n-2} \cap A_{n-1} \cap A_n|] \\ &\quad \vdots \\ &\quad + (-1)^{n-1} |A_1 \cap A_2 \cap \dots \cap A_{n-1} \cap A_n| \end{aligned}$$

Formulas MF27 does not give you

The syllabus expects you to use these, but the paper does not print them.

ALGEBRA

Quadratic formula and discriminant

for $ax^2 + bx + c = 0$. The three signs of the discriminant give two real roots, two equal roots and no real roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad b^2 - 4ac > 0, = 0, < 0$$

Modulus inequalities where $b > 0$

$$|x - a| < b \iff a - b < x < a + b$$

$$|x - a| > b \iff x < a - b \text{ or } x > a + b$$

Laws of logarithms $y = a^x$ is equivalent to $x = \log_a y$

$$\log_a xy = \log_a x + \log_a y, \quad \log_a \frac{x}{y} = \log_a x - \log_a y, \quad \log_a x^r = r \log_a x, \quad \log_a b = \frac{\log_c b}{\log_c a}$$

COORDINATE GEOMETRY

Equation of a circle centre (a, b) and radius r

$$(x - a)^2 + (y - b)^2 = r^2$$

SEQUENCES AND SERIES

Arithmetic series where l is the last term

$$u_n = a + (n - 1)d, \quad S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2} (a + l)$$

Geometric series $r \neq 1$. The sum to infinity exists only when $|r| < 1$

$$u_n = ar^{n-1}, \quad S_n = \frac{a(1 - r^n)}{1 - r}, \quad S_\infty = \frac{a}{1 - r}$$

n th term from the sum to n terms

$$u_n = S_n - S_{n-1}$$

TRIGONOMETRY

Pythagorean identities

$$\sin^2 A + \cos^2 A \equiv 1, \quad 1 + \tan^2 A \equiv \sec^2 A, \quad 1 + \cot^2 A \equiv \operatorname{cosec}^2 A$$

R formulae for $a, b > 0$, where $R = \sqrt{a^2 + b^2}$ and $\tan \alpha = b/a$

$$a \sin \theta + b \cos \theta = R \sin(\theta + \alpha), \quad a \cos \theta + b \sin \theta = R \cos(\theta - \alpha)$$

Small angle approximations x in radians

$$\sin x \approx x, \quad \cos x \approx 1 - \frac{1}{2}x^2, \quad \tan x \approx x$$

DIFFERENTIATION

Standard derivatives x in radians for the trigonometric functions

$$\frac{d}{dx}x^n = nx^{n-1}, \quad \frac{d}{dx}e^x = e^x, \quad \frac{d}{dx}\ln x = \frac{1}{x}$$
$$\frac{d}{dx}\sin x = \cos x, \quad \frac{d}{dx}\cos x = -\sin x, \quad \frac{d}{dx}\tan x = \sec^2 x$$

Chain, product and quotient rules

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}, \quad \frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}, \quad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

INTEGRATION

Standard integrals $n \neq -1$, arbitrary constants omitted. For $(ax + b)$ in place of x , divide by a

$$\int x^n dx = \frac{x^{n+1}}{n+1}, \quad \int e^x dx = e^x, \quad \int \sin x dx = -\cos x, \quad \int \cos x dx = \sin x, \quad \int \sec^2 x dx = \tan x$$

Integral of $1/x$ arbitrary constants omitted. This is the case $n = -1$ that the x^n rule leaves out

$$\int \frac{1}{x} dx = \ln |x|, \quad \int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b|$$

Integrals of $\sin^2 x$, $\cos^2 x$ and $\tan^2 x$

arbitrary constants omitted. Rewrite with $\cos 2A$ from MF27, or $\tan^2 x = \sec^2 x - 1$, first

$$\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4}, \quad \int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4}, \quad \int \tan^2 x dx = \tan x - x$$

Integrals involving $f'(x)$ and $f(x)$ $n \neq -1$, arbitrary constants omitted

$$\int f'(x) [f(x)]^n dx = \frac{[f(x)]^{n+1}}{n+1}, \quad \int \frac{f'(x)}{f(x)} dx = \ln |f(x)|, \quad \int f'(x) e^{f(x)} dx = e^{f(x)}$$

Integration by parts

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Volume of revolution

$$V = \pi \int_a^b y^2 dx \text{ (about the } x\text{-axis)}, \quad V = \pi \int_c^d x^2 dy \text{ (about the } y\text{-axis)}$$

VECTORS

Magnitude and scalar product where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}, \quad \mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

Projection and perpendicular distance where $\hat{\mathbf{n}}$ is a unit vector

$$\text{length of projection of } \mathbf{a} \text{ on } \hat{\mathbf{n}} = |\mathbf{a} \cdot \hat{\mathbf{n}}|, \quad \text{perpendicular component} = |\mathbf{a} \times \hat{\mathbf{n}}|$$

Equations of lines and planes

a line through the point with position vector $\mathbf{a}$ in direction $\mathbf{d}$, and a plane with normal $\mathbf{n}$

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}, \quad \mathbf{r} \cdot \mathbf{n} = D, \quad n_1 x + n_2 y + n_3 z = D$$

Cartesian equation of a line the line through (a_1, a_2, a_3) with direction vector $\mathbf{d}$

$$\frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$$

Angle between a line and a plane $\mathbf{d}$ is the direction of the line and $\mathbf{n}$ is the normal to the plane

$$\sin \theta = \frac{|\mathbf{d} \cdot \mathbf{n}|}{|\mathbf{d}||\mathbf{n}|}$$

Angle between two lines or two planes

$\mathbf{d}_1$ and $\mathbf{d}_2$ are the directions of the lines, $\mathbf{n}_1$ and $\mathbf{n}_2$ the normals to the planes. The modulus gives the acute angle

$$\cos \theta = \frac{|\mathbf{d}_1 \cdot \mathbf{d}_2|}{|\mathbf{d}_1||\mathbf{d}_2|}, \quad \cos \theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{|\mathbf{n}_1||\mathbf{n}_2|}$$

Distance from a point to a plane or a line

$\mathbf{b}$ is the position vector of the point. You can also find the foot of the perpendicular, then the distance between the two points

$$\text{plane } \mathbf{r} \cdot \mathbf{n} = D : \frac{|\mathbf{b} \cdot \mathbf{n} - D|}{|\mathbf{n}|}, \quad \text{line } \mathbf{r} = \mathbf{a} + \lambda \mathbf{d} : \frac{|(\mathbf{b} - \mathbf{a}) \times \mathbf{d}|}{|\mathbf{d}|}$$

COMPLEX NUMBERS

Modulus and conjugate

$$z = x + iy, \quad |z| = \sqrt{x^2 + y^2}, \quad z^* = x - iy, \quad zz^* = |z|^2$$

Argument

find the acute angle α first, then place $\arg z$ by the quadrant of z on the Argand diagram

$$\tan \alpha = \left| \frac{y}{x} \right|, \quad -\pi < \arg z \leq \pi$$

PROBABILITY AND STATISTICS

Probability rules for independent events, $P(A \cap B) = P(A)P(B)$

$$P(A') = 1 - P(A), \quad P(A \cup B) = P(A) + P(B) - P(A \cap B), \quad P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Permutations

the second is the number of ways to arrange n distinct objects in a circle

$${}^n P_r = \frac{n!}{(n-r)!}, \quad \text{in a circle: } (n-1)!$$

Expectation and variance of a discrete random variable

$$E(X) = \sum x P(X = x), \quad \text{Var}(X) = E(X^2) - [E(X)]^2$$

Linear combinations the last result needs X and Y to be independent

$$E(aX + b) = aE(X) + b, \quad \text{Var}(aX + b) = a^2\text{Var}(X)$$

$$E(aX + bY) = aE(X) + bE(Y), \quad \text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$$

Sample mean $\bar{x}$ is the unbiased estimate of the population mean

$$E(\bar{X}) = \mu, \quad \text{Var}(\bar{X}) = \frac{\sigma^2}{n}, \quad \bar{x} = \frac{\sum x}{n} = \frac{\sum(x - a)}{n} + a$$

Distribution of the sample mean

exactly for a normal population, approximately for large n (for example $n \geq 30$) by the Central Limit Theorem

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Unbiased variance from coded data use when the data are summarised as $\sum(x - a)$ and $\sum(x - a)^2$

$$s^2 = \frac{1}{n - 1} \left(\sum(x - a)^2 - \frac{(\sum(x - a))^2}{n} \right)$$

Standardising

If $X \sim N(\mu, \sigma^2)$ then $Z \sim N(0, 1)$. The second is the test statistic for a population mean, using s in place of σ for a large sample when σ is unknown

$$Z = \frac{X - \mu}{\sigma}, \quad Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

Regression line of x on y MF27 prints only the line of y on x

$$x - \bar{x} = b'(y - \bar{y}), \quad b' = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(y - \bar{y})^2}$$